

# Vortex Lattice Structures of a Bose-Einstein Condensate in a Rotating Triangular Lattice Potential

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We study the vortex pinning effect in a Bose-Einstein Condensate in the presence of a rotating lattice potential by numerically solving the time-dependent Gross-Pitaevskii equation. We consider a triangular lattice potential created by blue-detuned laser beams. By rotating the lattice potential, we observe a transition from the Abrikosov vortex lattice to the pinned vortex lattice. We investigate the transition of the vortex lattice structure by changing conditions such as angular velocity, strength, and lattice constant of a rotating lattice potential. Our simulation results clearly show that the lattice potential has a strong vortex pinning effect when the vortex density coincides with the density of the pinning points.

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## I. INTRODUCTION

Quantized vortices are one of the most characteristic manifestations of superfluidity associated with a Bose-Einstein condensate (BEC) in atomic gases [1]. Formations of triangular Abrikosov vortex lattice in Bose condensates have been clearly observed by rotating anisotropic trap potentials [2, 3, 4]. Microscopic mechanism of the vortex lattice formation has been extensively studied both analytically and numerically using the Gross-Pitaevskii equation for the condensate wavefunction [5, 6, 7, 8, 9, 10, 11, 12].

Another interesting development in ultracold atomic gases is creating periodic potential using optical lattices [13]. The studies of a BEC in optical lattices have found a lot of interesting phenomenon, such as the transition from the superfluid to Mott insulator phase [14].

More recently, combining the above two systems, i.e., rotating BEC and optical lattices has attracted growing attention. In particular, the vortex phase diagrams of a BEC in a rotating optical lattice potential have been theoretically studied, since one expects structural phase transition of vortex lattice structures due to vortex pinning [15, 16, 17, 18]. In the condensed matter systems such as superconductors, the vortex pinning due to impurities and defects in solids have been extensively investigated [19, 20]. In the atomic condensates, rotating optical lattices have been experimentally realized at JILA group, making use of a laser beam passing through a rotating mask [21]. This experiment observed the structural phase transition of vortex lattice structures in rotating Bose condensated gases due to vortex pinning by the laser beam. In this system, one can control the pinning parameters by changing conditions such as angular velocity, strength, and lattice constant of rotating optical lattices. This allows one to investigate vortex pinning effect in a quantitative manner.

In this paper, stimulated by the JILA experiment, we study the vortex pinning effect by numerically solving the

time-dependent Gross-Pitaevskii equation. We observe a structural phase transition of vortex lattice structures of a BEC in a rotating triangular lattice potential created by blue-detuned laser beams.

## II. FORMULATION

We consider a Bose-condensated gas trapped in a harmonic potential and a rotating periodic potential with an angular velocity  $\Omega$ . The dynamics of the condensate is described by the time-dependent Gross-Pitaevskii equation. Assuming a pancake-shaped trap, we use a two-dimensional Gross-Pitaevskii equation. In the rotating frame, the Gross-Pitaevskii equation is given by (for a detailed derivation, see for example Ref. [7])

$$(i - \gamma)\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\mathbf{r}) + g|\psi(\mathbf{r}, t)|^2 - \Omega L_z \right] \psi(\mathbf{r}, t). \quad (1)$$

Here,  $V_{\text{ext}}(\mathbf{r}) = m\omega_0^2(x^2 + y^2)/2 + V_{\text{lattice}}(\mathbf{r})$  describes the total external potential,  $L_z = -i\hbar(x\partial/\partial y - y\partial/\partial x)$  denotes the  $z$  component of the angular momentum operator,  $g = 4\pi\hbar^2 a_s/m$  is the strength of interaction with  $a_s$  being the  $s$ -wave scattering length, and  $\gamma$  is the phenomenological dissipative parameter [22, 23, 24, 25, 26]. The lattice potential created by blue-detuned laser beams arranged in the lattice geometry is expressed as

$$V_{\text{lattice}}(\mathbf{r}) = \sum_{n_1, n_2} V_0 \exp \left[ -\frac{|\mathbf{r} - \mathbf{r}_{n_1, n_2}|^2}{(\sigma/2)^2} \right], \quad (2)$$

where  $\mathbf{r}_{n_1, n_2} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2$  describes lattice pinning points and  $V_0$  describes the strength of the laser beam. We consider the triangular lattice geometry with two lattice unit vectors given by  $\mathbf{a}_1 = a(1, 0)$ ,  $\mathbf{a}_2 = a(-1/2, \sqrt{3}/2)$ , where  $a$  is the lattice constant.

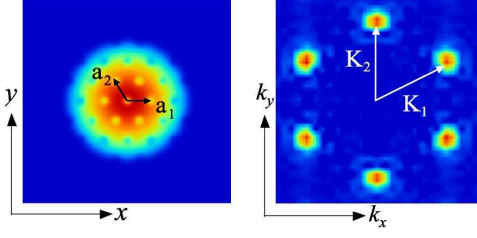


FIG. 1: Density profile (left) and Structure factor profile (right) of a BEC in a triangular lattice potential without rotation by setting  $\Omega = 0$ . The lattice potential geometry is triangular lattice at  $a/a_{\text{ho}} = 2.2$  and  $V_0/\hbar\omega_0 = 5.0$ .

Throughout this paper, we scale the length and energy by  $a_{\text{ho}} = \sqrt{\hbar/m\omega_0}$  and  $\hbar\omega_0$ . We set the dimensionless interaction strength as  $C = 4\pi a_s N/R = 1000$ , where  $N$  is the total particle number and  $R$  is size of the condensate along the  $z$ -axis direction [7], and the width of the laser beam as  $\sigma/a_{\text{ho}} = 0.65$ . In our parameter set, the healing length is  $\xi/a_{\text{ho}} = 0.12$ . We numerically solve the Gross-Pitaevskii equation in Eq. (1) using Fast-Fourier-Transform (FFT) technique.

We first determine the ground-state condensate wavefunction without rotation by setting  $\Omega = 0$  in Eq. (1). Starting with this wavefunction as a initial state, we numerically evolve the Gross-Pitaevskii equation with a rotating lattice potential with a fixed angular velocity  $\Omega$ , until the system relaxes into equilibrium. We investigate the vortex lattice structure from the equilibrium condensate density profile  $n(\mathbf{r}) = |\psi(\mathbf{r})|^2$ . In order to quantify the vortex lattice structure, we calculate the density structure factor defined by

$$S(\mathbf{k}) = \int d\mathbf{r} n(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}}. \quad (3)$$

The structure factor given in Eq. (3) contains information about the periodicity of the condensate density. For triangular lattice geometry,  $S(\mathbf{K})$  exhibits periodic peaks of the regular hexagonal geometry. In order to distinguish between the Abrikosov lattice and the pinned vortex lattice, we calculate the peak intensity of the structure factor  $|S(\mathbf{K})|$  at the lattice pinning point, namely,  $\mathbf{K}_1 = 2\pi/a(1, 1/\sqrt{3})$ ,  $\mathbf{K}_2 = 2\pi/a(0, 2/\sqrt{3})$ . In addition to the structure factor, we also calculate the lattice potential energy defined by

$$E_{\text{lattice}} = \int d\mathbf{r} \psi^*(\mathbf{r}) V_{\text{lattice}}(\mathbf{r}) \psi(\mathbf{r}), \quad (4)$$

which can also be used to quantify the transition of the vortex structure. As we see below, the vortex pinning is signified by a marked decrease of  $E_{\text{lattice}}$ .

### III. SIMULATION RESULTS

In this section, we show our numerical simulation results for vortex lattice structures of a BEC in a rotating

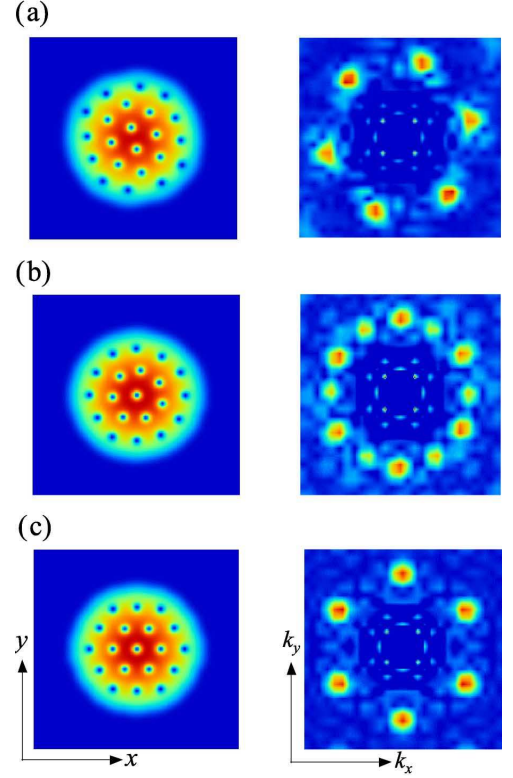


FIG. 2: Density profiles (left) and Structure factor profiles (right) in equilibrium state after rotating condensates with a fixed angular velocity  $\Omega/\omega_0 = 0.70$  at  $a/a_{\text{ho}} = 2.2$ . The figures show vortex lattice structures for different values of the strength of a triangular lattice potential; (a)  $V_0/\hbar\omega_0 = 0.1$ , (b)  $V_0/\hbar\omega_0 = 0.3$ , (c)  $V_0/\hbar\omega_0 = 0.5$ .

triangular lattice potential. Fig. 1 shows the equilibrium condensate density profile and the density structure factor profile in the presence of a triangular lattice potential without rotation.

Fixing the lattice constant with  $a/a_{\text{ho}} = 2.2$ , we investigate the transition of the vortex lattice structure by changing the strength  $V_0$  for various angular velocities  $\Omega$ . Fig. 2 shows the density profiles and the structure factor profiles in equilibrium state after rotating condensates with a fixed angular velocity  $\Omega/\omega_0 = 0.70$ . One can see that for weak lattice potential  $V_0/\hbar\omega_0 = 0.1$  (Fig. 2 (a)), vortices form the Abrikosov lattice, which is incommensurate with the triangular lattice potential. By slightly increasing the lattice strength as  $V_0/\hbar\omega_0 = 0.3$  (Fig. 2 (b)), vortices start to being partially pinned by the triangular lattice potential. For strong lattice potential  $V_0/\hbar\omega_0 = 0.5$  (Fig. 2 (c)), all vortices are pinned by the triangular lattice potential. We thus observed a transition of the vortex lattice structure from the Abrikosov lattice to the pinned lattice.

In Fig. 3, we plot  $|S(\mathbf{K})|$  and  $E_{\text{lattice}}$  against the lattice strength  $V_0$ . As shown in Fig. 3 (a), for  $\Omega/\omega_0 = 0.70$ , the lattice potential energy  $E_{\text{lattice}}$  decreases gradually, which indicates that vortices are partially pinned,

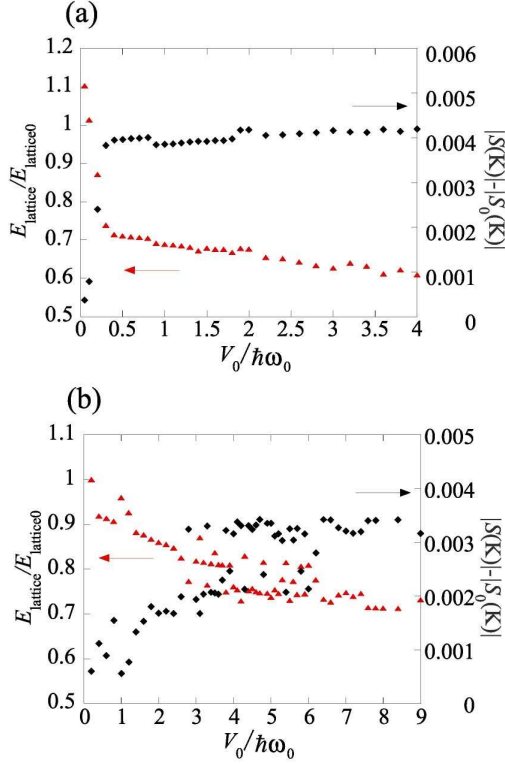


FIG. 3: Lattice potential energy ( $\blacktriangle$ ) and peak intensity of the structure factor at the lattice pinning point ( $\blacklozenge$ ) with  $a/a_{ho} = 2.2$  corresponding to the angular velocity; (a)  $\Omega/\omega_0 = 0.70$ , (b)  $\Omega/\omega_0 = 0.55$ . Here  $E_{lattice0}$  and  $|S_0(\mathbf{K})|$  is the lattice energy and the peak intensity of the structure factor at the lattice pinning point of the ground state ( $\Omega = 0$ ) for each lattice strength.

and reaches constant when all vortices are pinned for  $V_0/\hbar\omega_0 \geq 0.4$ . Correspondingly the structure factor  $|S(\mathbf{K})|$  increases gradually and finally becomes constant when all vortices are pinned. From these results, together with directly looking at the condensate density profile, we conclude that the structural phase transition occurs at  $V_0/\hbar\omega_0 = 0.4$ , which we define as the strength for the structural phase transition  $V_c/\hbar\omega_0$ . Fig. 3 (b) shows the analogous result for  $\Omega/\omega_0 = 0.55$ . In this case, there is an intermediate domain where the Abrikosov lattice and the pinned lattice coexist. In this domain, the vortex lattice structure cannot be categorically determined because of the competition between the vortex-vortex interaction and the lattice pinning effect.

From these results, we map out the phase diagrams of vortex lattice structures against  $\Omega$  and  $V_0$ , as we plot in Fig. 4 (a). The lower domain is the Abrikosov lattice domain, while the upper domain is the pinned lattice domain. The intermediate domain represents coexisting state of the Abrikosov lattice and the pinned lattice. Looking at the phase boundary of the pinned lattice domain in Fig. 4 (a) as  $V_c = V_c(\Omega)$ , we find that  $V_c$  takes a minimum value as a function of  $\Omega$ , which we define as the minimum pinning strength  $V_{c,min} = V_c(\Omega = \Omega_{min})$ . From

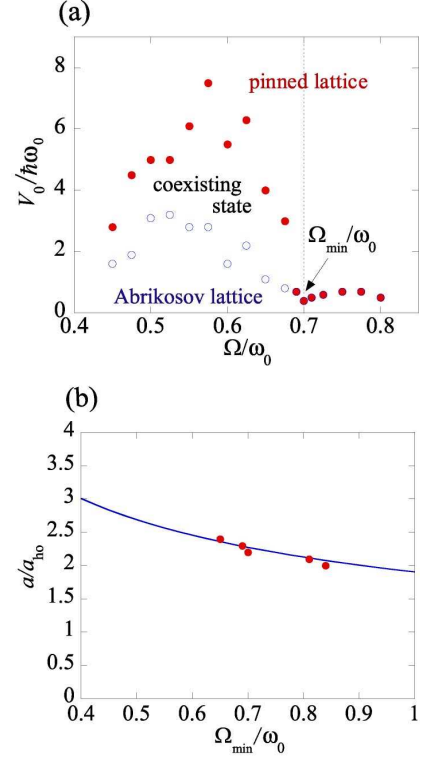


FIG. 4: (a) Phase diagrams for vortex lattice structures for  $a/a_{ho} = 2.2$ . (b) Lattice constant  $a$  against  $\Omega_{min}$  giving the minimum pinning strength. The solid line represents Eq. (5).

Fig. 4 (a) for  $a/a_{ho} = 2.2$ , we find that  $V_{c,min}/\hbar\omega_0 \approx 0.4$  and  $\Omega_{min}/\omega_0 \approx 0.70$ .

The dependence of minimum pinning lattice strength  $V_{c,min}$  on the lattice constant  $a$  can be understood as follows. When vortices form the triangular lattice and undergo rigid rotation with angular velocity  $\Omega$ , the lattice constant is expressed as a function of angular velocity by

$$\frac{a_v(\Omega)}{a_{ho}} = \sqrt{\frac{2}{\sqrt{3}} \frac{2\pi}{2\Omega/\omega_0}}. \quad (5)$$

In Fig. 4 (b), we plot lattice constant  $a$  against  $\Omega_{min}$ . We find that the formula in Eq. (5) well fits the numerical data. This means that the pinning strength  $V_c$  takes minimum value when  $a_v(\Omega)$  matches to the lattice constant  $a$ . When this “matching relation” is satisfied, weak lattice potential has a stronger pinning effect than vortex-vortex interaction, which leads to a sharp transition of the vortex lattice structure.

In the case when this “matching relation” is satisfied, one may presume that the vortex lattice array match the lattice potential array without vortex pinning effect. However, from Fig. 4 (a), we find that minimum pinning lattice strength  $V_{c,min}$  does not fall to zero, but takes a finite value. Actually in obtaining the phase diagram in Fig. 4 (a), we solved the Gross-Pitaevski equation starting with the initial ground state wavefunction without

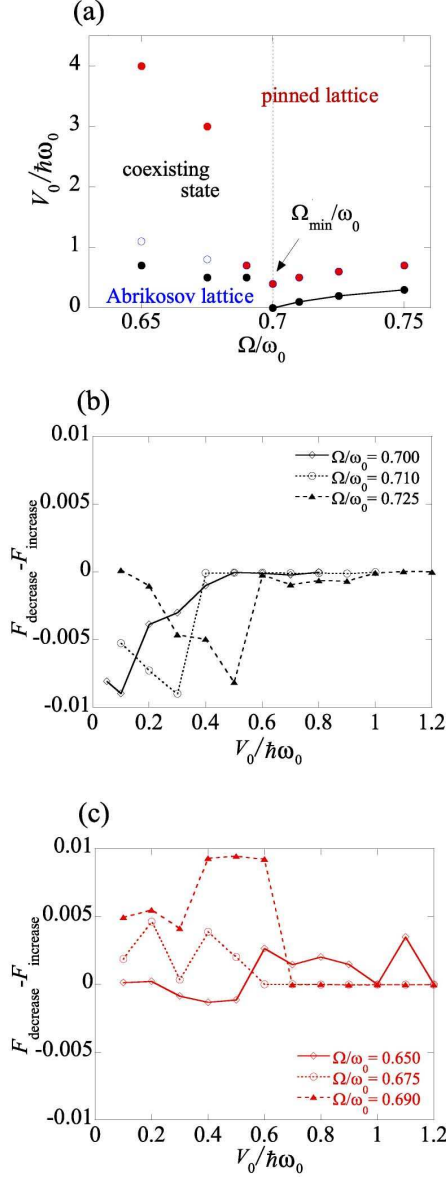


FIG. 5: (a) Phase diagrams for vortex lattice structures for  $a/a_{ho} = 2.2$  by increasing  $V_0$  or decreasing  $V_0$  from pinned lattice domain ( $\blacklozenge$ ). At  $\Omega \geq \Omega_{\min}$ , phase boundary of actually the pinned lattice phase is solid line. (b) Total free energy  $F$  with  $a/a_{ho} = 2.2$  at  $\Omega/\omega_0 = 0.70$  ( $\diamond$ ),  $\Omega/\omega_0 = 0.71$  ( $\circ$ ) and  $\Omega/\omega_0 = 0.725$  ( $\blacktriangle$ ). Here  $F_{\text{increase}}$  is in the case of increase of  $V_0$ , while  $F_{\text{decrease}}$  is in the case of decrease of  $V_0$  from pinned lattice domain. (c) Total free energy  $F$  with  $a/a_{ho} = 2.2$  at  $\Omega/\omega_0 = 0.65$  ( $\diamond$ ),  $\Omega/\omega_0 = 0.675$  ( $\circ$ ) and  $\Omega/\omega_0 = 0.69$  ( $\blacktriangle$ ).

vortices. As evolving the Gross-Pitaevski equation, vortices are nucleated, forming vortex lattices. In this dynamical process, it may be possible that vortices relax into metastable configurations, and this may be the reason why  $V_{c,\min}$  takes a finite value. In order to investigate this in more detail, we solve the Gross-Pitaevski equation starting with a perfectly pinned vortex lattice at large enough  $V_0$ . We then slowly decrease  $V_0$ . In Fig. 5 (a),

we plot the phase boundary of the pinned lattice phase obtained in this way, together with phase boundaries previously shown in Fig. 4 (a). We find that at  $\Omega = \Omega_{\min}$ , the perfectly pinned vortex lattices is stable down to infinitesimally small lattice potential  $V_0 \rightarrow 0$ . In order to see whether the pinned state obtained here has the lower energy than the tilted Abrikosov lattice obtained in the previous calculation, we calculate the total free energy

$$F = \left\langle \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\mathbf{r}) + g|\psi(\mathbf{r}, t)|^2 - \Omega L_z \right] \psi(\mathbf{r}, t) \right\rangle. \quad (6)$$

In Figs. 5 (b) and (c), we plot the free energy  $F$  for  $a/a_{ho} = 2.2$  against  $V_0$ . Here,  $F_{\text{increase}}$  is the free energy obtained by solving the Gross-Pitaevski equation starting with non-vortex initial state and increasing  $V_0$ , while  $F_{\text{decrease}}$  is the free energy obtained by decreasing  $V_0$  from pinned lattice domain. We make the separate plots for  $\Omega \geq \Omega_{\min}$  and  $\Omega < \Omega_{\min}$  in Figs. 5 (b) and (c), respectively. One can see from Fig. 5 (b) that, at  $\Omega = \Omega_{\min}$ ,  $F_{\text{decrease}} < F_{\text{increase}}$  holds down to  $V_0 \rightarrow 0$ . This means that the Abrikosov lattice found for  $0 < V_0 < V_{c,\min}$  in the phase diagram in Fig. 4 (a) is actually metastable. Similarly for  $\Omega > \Omega_{\min}$ , we find a finite region where the Abrikosov lattice is metastable. However, for  $\Omega < \Omega_{\min}$ , we find  $F_{\text{decrease}} > F_{\text{increase}}$  as shown in Fig. 5 (c). This means that the pinned vortex lattice does not have the lowest energy in the weak lattice domain.

From the above results, we found that property of vortex pinning is quite different for  $\Omega \geq \Omega_{\min}$  and  $\Omega < \Omega_{\min}$ . For  $\Omega \geq \Omega_{\min}$ , the vortex lattice structure exhibits sharp transition. In contrast, for  $\Omega < \Omega_{\min}$ , there is no sharp transition, but the vortex lattice structure exhibits crossover through the intermediate coexisting region.

#### IV. SUMMARY AND DISCUSSION

In summary, we have studied the vortex pinning effect by observing the structural phase transition of vortex lattice structures of a Bose-Einstein condensate in a rotating triangular lattice potential. We observed the transition of the vortex lattice structure from the Abrikosov vortex lattice to the pinned lattice where all vortices are pinned to lattice points. The transition is determined depending on the competition between strength and density of the lattice potential, and vortex density and vortex-vortex interaction. From our simulation, we found that the lattice potential has a strong vortex pinning effect when  $a_v(\Omega)$  matches to the lattice constant  $a$  of the triangular lattice potential, which means that the vortex density coincides the density of pinning points. In the case when this “matching relation” is satisfied, we observed that minimum pinning lattice strength  $V_{c,\min}$  takes a finite value. By investigating the structural transition in more detail, we found that for  $\Omega \geq \Omega_{\min}$ , there are regions where the Abrikosov lattice is metastable, while for

$\Omega < \Omega_{\min}$ , the pinned vortex lattice phase is metastable state. From the above results, we found that property of vortex pinning is quite different for  $\Omega \geq \Omega_{\min}$  and  $\Omega < \Omega_{\min}$ . For  $\Omega \geq \Omega_{\min}$ , the vortex lattice structure exhibits sharp transition. In contrast, for  $\Omega < \Omega_{\min}$ , there is no sharp transition, but the vortex lattice structure exhibits crossover through the intermediate coexisting region.

In a separate paper, we will discuss the pinning effect of a rotating square lattice potential. We will show that the property of vortex pinning is quite different from the triangular lattice and more complicated for a rotating

square lattice potential.

## V. ACKNOWLEDGMENTS

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